# Meteoric Flux and Density Fields about an Infinitesimal Attractive Center Generated by a Stream Monoenergetic and Monodirectional at Infinity

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N64 32382

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*Abstract.* The meteoric field structure theory previously developed for meteoric streams monoenergetic and monodirectional at infinity is applied to the problem of a meteoric stream incident upon an infinitesimal attractive center. Flux and density contours about the center are explicitly obtained for a particle speed at infinity of 2 km/sec as an example of a method developed to provide flux and density contours for any speed. An incident stream five to ten earth diameters in width results in an order of magnitude enhancement of flux at points downstream from the attractive center. The flux patterns for any energy can be derived from a universal flux plot in terms of a dimensionless parameter $\lambda = yr$, where $y = V_\infty^2/\gamma M$, and $r$ is distance in earth (or center) radii. This universal flux plot is shown.

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*Introduction.* The broad objectives of this paper are the development of the detailed flux and density meteoric fields generated by an incident meteoric stream, which at infinity is monoenergetic and monodirectional, interacting with an attractive center of finite extent. The fields for an infinitesimal attractive center are also obtained; these are simpler than for a finite center by virtue of the fact that no screened zones can appear.

We feel that this is an important case for attractive centers in the solar system other than the sun, relative to which few meteors possess hyperbolic energies. For bodies of planetary size the situation is different; that is, in some local regions, they do provide the dominant gravitational field; relative to them, meteoric bodies do possess hyperbolic energies. Thus, in essence, we are assuming that a meteoric stream, whose trajectory is determined almost everywhere by the solar field, in the near vicinity of a planet (e.g., the earth) can nevertheless be usefully approximated as a stream which relative to the planet is monodirectional and monoenergetic at infinity.

In developing the hypothesis we are not asserting that the capture of, and captured, meteoric particles may not be an existent and important phenomenon; rather, we will demonstrate that monoenergetic, monodirectional streams at infinity constitute one mechanism for the realization of meteoric field patterns exhibiting large and abrupt variations in flux and density. That such variations exist appears to be well established; furthermore, they are required by several theories for the explanation of a wide range of phenomena. Direct counting of meteoric impact rates by means of satellite experiments reveals, within a few hundred kilometers of the earth, flux concentrations [*Singer*, 1961, 1963; *Bohn et al.*, 1950; *Rushol*, 1963; *Dubin*, 1956; *Berg and Meredith*, 1956; *Gallagher and Eshleman*, 1960] of dust that are indicated, on the basis of some zodiacal light–solar *F* corona data, to be orders of magnitude greater than those far from the earth. Moreover, observed impact rates vary rapidly within periods of hours [*LaGow and Alexander*, 1960; *Alexander et al.*, 1961; *Dubin*, 1960; *Dubin et al.*, 1964; *Dubin and McCracken*, 1962] but otherwise behave as might be expected from concentrated streams.

Possibly of special interest, relative to the application of the theory to be developed here, is the case of dust moving about the sun at a distance of about 1 AU. The speed of these particles relative to the earth would be quite small and, indeed, would be treated by our theory as possessing speeds relative to the earth, at infinity, of a few kilometers per second at the most. Their observed relative velocities would be due principally to a conversion of gravitational geopotential energy. It is precisely these very slow particles which give rise to the most complex meteoric fields about a finite earth. These, of course, are in addition to the small particles of approximately normal distribution in speed reported by *Eshleman and Gallagher* [1962]. Their existence and field patterns might be important in discussions of (1) the zodiacal light [*James*, 1963], (2) the concentration of fine Ni particles within noctilucent clouds where densities relative to space outside the cloud are greater by orders of magnitude [*Soberman*, 1963; *Witt et al.*, 1963], (3) the concentration of condensation nuclei for ordinary rain clouds, and (4) the variation of concentration of dust responsible for the radiance [*Barber*, 1962] of twilight sky. This last phenomenon (4) apparently is influenced by the position of the moon —an effect formerly thought impossible through the influence of the lunar gravitational field, but which, now viewed in terms of the field patterns for very slow particles (predicted by the theory to follow), may be a significant factor. Furthermore, it is hoped that the results obtained in this study can be profitably applied to the analysis of the data to be obtained from the NASA meteoroid measurement satellite, and especially to the data from satellite meteorite observations made farther from the earth which will measure the energy of the impinging particles.

As in most physical theories, our development has some unrealistic features: we assume an infinitely broad, infinitely persisting, monoenergetic-monodirectional meteoric stream at infinity, incident upon a single attractive center, and derive the flux and density contours for various energies which would result in such a situation. The probable importance of other forces for very fine particles (i.e., radiation pressure, Poynting-Robertson effect, and Coulomb drag) is also recognized; therefore, in situations where these are also prominent, our present development can be considered as preliminary.

Fundamental to this development are the concepts of density, current, and flux. Here, density is simply the number of particles per unit volume. The current at any point is a vector tangent to the particle trajectory, passing through the given point, and pointing in the direction of particle motion; the magnitude of the current vector is chosen equal to the number of particles moving along the trajectory which cross a unit area normal to the trajectory during unit time. The magnitude of the current is the product of the density and particle speed at the point. In situations where a given point may be threaded by trajectories passing in two or more directions, the current concept loses its usefulness. In such cases it may be generalized to the concept of flux which is defined to be the total path length generated or swept out per unit volume in unit time. Flux is a scalar quantity, obtained by adding the magnitudes of the various current vectors passing through the point; in cases where only one trajectory is possible, flux obviously becomes the magnitude of the current.

For the reader's convenience in understanding this paper, we present a very brief discussion of the theory involved in forming the basic equation. The derivation of this equation and a great deal of peripheral material is treated in detail by *Shelton et al.* [1964].

If the value of the flux at infinity between impact parameters $a$ and $a + da$ is given by $J(\infty)$, the value for the flux between $r$ and $r + dr$ is given by

$$-J(\bar{r}) \cdot d\bar{A} = J(\bar{r})\, dA \cos\alpha = 2\pi a\, da\, J(\infty)$$

Writing the element of area $dA$ in spherical coordinates as

$$dA = 2\pi r^2 \sin\theta\, d\theta$$

we have, after rearrangement,

$$\frac{J(r)}{J(\infty)} = \frac{a\,da}{r^2 \sin\theta \cos\alpha\, d\theta}$$

which is the basic equation used here for the development of the flux and density fields.

The impact parameter $a$ is obtained from the well-known orbit equation [*Goldstein*, 1959]

$$r = \frac{ya^2}{1 + \sqrt{1 + y^2a^2} \cos(\theta - \theta_k)}$$

where

$$y = V_\infty^2/\gamma M$$

$\gamma$ = gravitational constant.
$M$ = mass of center of force.

The function $\cos\alpha$ is obtained directly from the conservation of angular momentum equation

$$V(r) \sin\alpha = V_\infty a$$



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where $\alpha$ is the angle measured from the inward pointing radius vector to the directed trajectory defined to be positive in the counterclockwise sense.

*Method and mode of presentation.* Fundamentally this paper is simply a detailed application of the meteoric field structure theory [*Shelton et al.*, 1964] developed for monodirectional, monoenergetic streams at infinity, incident upon an attractive center. They have shown that, about an infinitesimal attractive center, the flux field $\phi(r, \theta)$ at any point specified in the plane by polar coordinates $r$ and $\theta$ with origin at the center is generally given by the addition of two currents. One current may be unscattered flux, the other scattered flux, or both currents may be scattered. The pertinent equations are sufficiently expressed in terms of an impact parameter $a$, the angle $\alpha$ between the radius vector and the tangent to one of the two trajectories threading every field point in the case of an infinitesimal attractive center, and $y$ defined by

$$y = V_\infty^2/\gamma M \tag{1}$$

where $\gamma$ is the gravitational constant, $M$ is the mass of the attractive center, and $V_\infty$ is the speed of the stream at infinity. If $\phi_\infty$ is the flux at infinity, then

$$\frac{\phi(r,\theta)}{\phi_\infty} = \frac{a_1}{r^2 \sin\theta \cos\alpha_1}\frac{da_+}{d\theta} + \frac{a_2}{r^2\sin\theta\cos\alpha_2}\frac{da_-}{d\theta} \tag{2}$$

where the $a$'s and $\alpha$'s are assigned (Table 1) according to whether the field point is in the upstream sector (sector 5) [*Hale and Wright*, 1964] where one current is unscattered and the other scattered, or in the downstream sector (sector 3)[1] where both currents scatter in different directions.

In Table 1

$$a_\pm = \frac{ry\sin\theta \pm \sqrt{r^2y^2\sin^2\theta + 4yr(1-\cos\theta)}}{2y} \tag{3}$$

[1] The sectors are numbered 5 and 3 to conform to the accompanying paper in which the discussion is extended to the case of the finite attractive center.

TABLE 1. Compatible Set of $a$'s and $\alpha$'s

| Sector | $a_1$ | $\alpha_1$ | $a_2$ | $\alpha_2$ |
|---|---|---|---|---|
| 5. Unscattered and scattered flux | $a_+$ | $\alpha_+(a_+)$ | $a_-$ | $\alpha_-(a_-)$ |
| 3. Scattered flux only | $a_+$ | $\alpha_-(a_+)$ | $a_-$ | $\alpha_-(a_-)$ |

$$\frac{da_\pm}{d\theta} = \frac{1}{2}\left[r\cos\theta + \frac{r^2y\cos\theta\sin\theta + 2r\sin\theta}{2ya_\pm - ry\sin\theta}\right] \tag{4}$$

$$\cos\alpha_\pm = \pm\left[\frac{r^2y^2 + 2ry - y^2a_\pm^2}{r^2y^2 + 2ry}\right]^{1/2} \tag{5}$$

where the $\alpha$'s are shown as functions of $a$'s, their associated impact parameters, the actual function being exhibited in (5).

The ultimate objective is to present flux and particle density data in the form of isoflux and isodensity contours about the attractive center. Certain general features of these plots require detailed explanation. One of these is the $\theta_{k\,\max}$ surface, referred to in the plane as the $\theta_{k\,\max}$ line. (This problem is axially symmetric throughout. The actual three dimensional field patterns can, in all cases, be generated by rotating the two-dimensional plots about on axis through the center and parallel to $V_\infty$.) For the whole continuum of trajectories about the attractive center, this surface is simply the locus of points of perigee. Before crossing the $\theta_{k\,\max}$ surface, a trajectory is unscattered; after crossing the surface, the trajectory is scattered (i.e., receding from the center). Both the contours and gradients of both flux and density are continuous upon crossing the $\theta_{k\,\max}$ surface; in the case of the finite scattering center, there are surfaces for which this is not true. Thus, for a stream, incident upon the earth with a particle speed $\geq 5$ km/sec, the associated $\theta_{k\,\max}$ surface divides all space about the center into two sectors; i.e., this surface is the boundary between sector 1 and sector 2 [*Hale and Wright*, 1964].

Flux contours were obtained by first holding $\theta$ constant in (2) and calculating $\phi$ as a function of $r$ for a given value of $y$ (effectively, the energy). Performing this for various angles, a series of radial profiles was then plotted (a typical set of profiles is shown in Figure 1). Then, for a fixed value of $\phi$, pairs of $(r, \theta)$ values can easily be

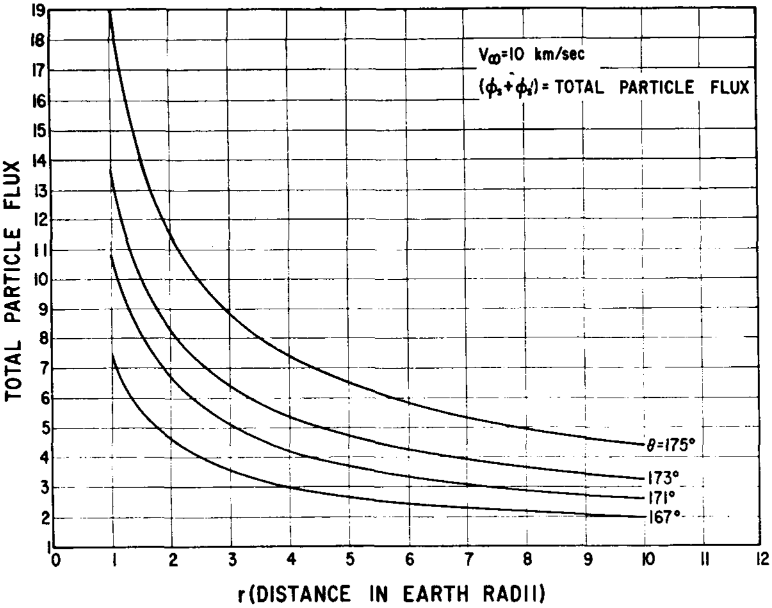


Fig. 1. Radial profiles of total particle flux for $V_\infty = 10^4$ m/sec.

read. The set of $(r, \theta)$ values for a given $\phi$ value defines a flux contour.

Once we have the isoflux contours, density contours can be found by dividing the flux values at the field by the stream speeds; i.e., $\phi$ is essentially the path length swept out in unit volume during unit time, or

$$\phi \equiv \sum_i \rho_i V_i \tag{6}$$

where the summation is over the currents traversing the unit volume. Each current, by carrying a cylinder of unit base of height $V_i$ and particle density $\rho_i$ through a unit volume during unit time, generates an amount of path length within the unit volume equal to $\rho_i V_i$. Since in our problem $V_i$ depends only on the location of the field points,

$$\phi = \sum_i \rho_i V_i = V \sum_i \rho_i$$

or

$$\rho(r) \equiv \sum \rho_i = \frac{\phi}{V(r)} \tag{7}$$

where the total particle density has been defined as the sum of the densities arising from the various currents.

*Flux through unit area as $\theta \to \pi$, $r \to \infty$.* It is obvious, upon inspection of (2), that the flux approaches infinite values as $\theta \to \pi$. This, however, is a trivial singularity since, upon

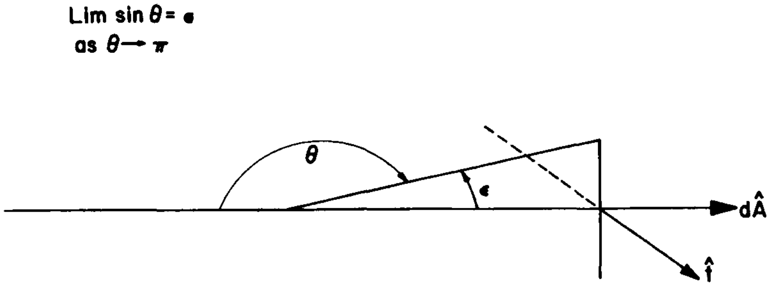


Fig. 2. Geometry in vicinity of $\theta = \pi$.

integration of the total flux, threading any finite area containing the axis ($\theta = \pi$) is always finite at any finite point.

In Figure 2, $\mathbf{t}$ is a unit vector along one of the trajectories threading the unit area $dA$, represented by the unit vector $d\mathbf{A}$. We define $\bar{I}_\pi$ to be the magnitude of the total current passing through $d\mathbf{A}$ and consider

$$\lim_{\theta\to\pi} \left\{\frac{2a}{r^2 \sin\theta \cos\alpha} \frac{da}{d\theta} \mathbf{t} \cdot d\mathbf{A}\right\} \equiv \bar{I}_\pi \equiv \frac{\bar{\phi}_\pi}{\phi_\infty} \quad (8)$$

where it is to be understood that $r$ exceeds by orders of magnitude the unit length, thereby justifying the replacement of the integration by simply multiplying the flux density by the projected area; the factor of 2 arises from the symmetry of particle trajectories near the axis (i.e., $\phi_S \to \phi_S'$ as $\theta \to \pi$, and barred quantities are averaged values over the surface element $d\mathbf{A}$). Using

$$\lim_{\theta\to\pi} \begin{cases} a \to \sqrt{2r/y} \\ da/d\theta \to -r/2 \end{cases}$$

$$\sin\theta = \sin(\pi - \theta) = \sin\epsilon = \epsilon$$

we have

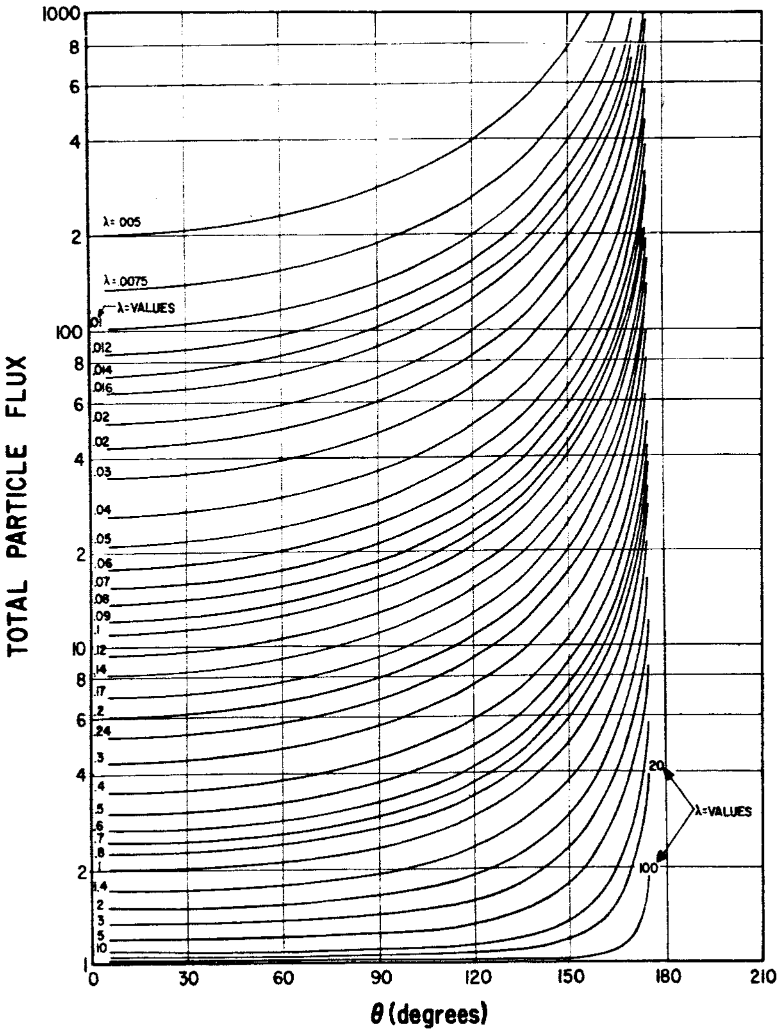


Fig. 3. Universal flux plots relative to unit monodirectional, monoenergetic incident flux at infinity about an infinitesimal attractive center in terms of parameter $\lambda \equiv yr$, where $y = V_\infty^2/\gamma M$, and $r$ = distance in earth radii.

$$\bar{I}_\pi \longrightarrow -2\sqrt{\frac{2r}{y}}\frac{r}{2}\frac{\mathbf{t}\cdot d\mathbf{A}}{r^2\epsilon\cos\alpha} \tag{9}$$

where the minus sign denotes diverging flux and $\cos\alpha$ is finite and monotonically approaching unity as $r \longrightarrow \infty$. Next, consider

$$1 \equiv |d\mathbf{A}| = \pi r^2 \sin^2(\pi - \theta) \simeq \pi r^2 \epsilon^2 \tag{10}$$

which yields

$$\epsilon \simeq 1/(r\sqrt{\pi})$$

Since $\mathbf{t}\cdot d\mathbf{A} = \cos\alpha$, and substituting for $\mathbf{t}\cdot d\mathbf{A}$ and $\epsilon$, we find

$$\bar{I}_\pi = \lim_{\theta\to\pi}\frac{\phi}{\phi_\infty} = -\sqrt{\frac{2r}{y}}\frac{r(r\sqrt{\pi})}{r^2}$$

$$= -\sqrt{\frac{2\pi r}{y}} = -\sqrt{\frac{2\pi r\gamma M}{V_\infty^{\ 2}}} \tag{11}$$

for flux through unit area on the axis.

For any angle $\theta \neq \pi$, and $r$ approaching infinity, one merely has to substitute in (2)

$$\lim_{\substack{r\to\infty\\ \theta\neq\pi}}\begin{cases} a_- \longrightarrow 0 & a_+ \longrightarrow r\sin\theta \\ \dfrac{da_-}{d\theta} \longrightarrow 0 & \dfrac{da_+}{d\theta} \longrightarrow r\cos\theta \\ \cos\alpha_- \longrightarrow -1 & \cos\alpha_+ \longrightarrow \cos\theta \end{cases}$$

to see that the contribution from scattered radiation (corresponding to $a_-$) vanishes at infinity as it should, leaving

$$\bar{I}_\theta \equiv \lim_{\substack{r\to\infty\\ \theta\neq\pi}}\frac{\Phi}{\phi_\infty} = \frac{r^2\sin\theta\cos\theta}{r^2\sin\theta\cos\alpha}\,\mathbf{t}\cdot d\mathbf{A} = \cos\theta \tag{12}$$

which is exactly what we would expect, this being the result for an undisturbed flux incident upon a unit area whose normal is inclined at direction $(\pi - \theta)$ with respect to the radiation stream.

*Universal flux plot for a monoenergetic-monodirectional stream incident on an infinitesimal attractive center.* Intuitively, one feels that with the exception of a scale factor the flux field pattern about an infinitesimal attractive center should be independent of energy. This will be shown to be true. At any point about an infinitesimal attractive center, the total flux density consists of two of three possible components, the pertinent pair depending upon in which of the two spherical sectors the point is located. Field points in the upstream sector have flux contribution from direct (unscattered) flux $\phi_D$ and from flux scattered into the sector from the other hemisphere $\phi_S'$ (scattered); field points in the downstream sector have flux contributions from flux scattered from the same hemisphere $\phi_S$ (scattered) and $\phi_S'$ (scattered). Thus, in general, the total flux

$$\phi = \sum_{i=S\text{ or }D,S'} |\phi_i|$$

$$= \sum\left|\frac{a_i}{r^2\sin\theta\cos\alpha_i}\left(\frac{da_i}{d\theta}\right)\right| \tag{13}$$

The quantities $ya$, $y(da/d\theta)$, and $\cos\alpha$ upon explicit exhibition seem to be functions of $\theta$ and $yr$. When we define

$$yr = \lambda \tag{14}$$

it follows that

TABLE 2. Interpretations of Universal Plot

| | $y$ = constant | $r$ = constant |
|---|---|---|
| If $\phi$ = constant | Line $\phi$ = constant in Figure 3 generates a constant $\phi$ surface (flux contour) in real space. | Line $\phi$ = constant in Figure 3 generates $y$ values which at the associated points $(r, \theta)$ would deliver total flux $\phi$. |
| If $\theta$ = constant | Line $\theta$ = constant in Figure 3 directly yields flux along radial direction in real space. | Line $\theta$ = constant in Figure 3 generates associated $y$ and $\phi$ values for point specified by $\theta$ = constant, $r$ = constant. |
| If $\lambda$ and $r$ | = constant, Figure 3 generates gnomonic plots, i.e., total flux on surfaces of spheres in real space. | |
| If $r\cdot y$ | = constant (both $r$ and $y$ vary inversely), the angle at which perigee occurs, $\theta_k$, is specified by $\cos\theta_k = -1/(1+\lambda)$. Cones with a common vertex at origin are generated in real space, on the surface of which trajectories, associated with the $y$ values determined from $y = \lambda/r$ and having impact parameters $a = r[1 + (2/ry)]^{1/2}$ [*Shelton et al.*, 1964], attain perigee. | |

$$\phi = \sum_i \left| \frac{ya_i}{y^2r^2 \sin\theta \cos\alpha_i} \frac{yda_i}{d\theta} \right| = \phi(\theta, \lambda) \quad (15)$$

Equation 15, under certain restrictions, permits the construction of a universal flux plot so named because from it the field about any infinitesimal attractive center can be generated for any energy. The results of *Shelton et al.* [1964] show that $\phi_S'$ contributes to the flux at every point and that, with the exception of sign-scattered flux being negative, one function describes both $\phi_D$ and $\phi_S$. Thus, we have attained a function of $\lambda$ and $\theta$ only, which represents the total flux at any point about an infinitesimal attractive center. We should note, however, that this cannot be done for the net flux where the sign of the contribution *is* important.

This function $\phi(\lambda, \theta)$ is displayed in Figure 3 plotted against $\theta$, with $\lambda$ being the parameter for a family of $\lambda$ = constant curves. For a given angle and specified $y$ (the energy and attractive center strength) and distance, we can readily read off the total flux at that point relative to unit flux at infinity. Alternatively, we may want to know for a given flux corresponding to a certain energy at what distance from the center along a given direction (a radial line) will the flux be the same for another energy or attractive center strength. This question led to the idea of a universal plot and is easily answered by considering

$$\lambda_i = y_i r_i = \frac{V_{i\infty}{}^2 r_i}{\gamma M_i} = \frac{V_{j\infty}{}^2 r_j}{\gamma M_j} = y_j r_j = \lambda_j$$

or we might wish to know for a given distance from the center and for a specified energy (these together determine $\lambda$) the minimum value of $\theta$ for which the flux exceeds a stated value.

Figure 3 includes the following range for an earth mass attractive center ($R_E$ is the earth's radius):

$$0.005 \leq \lambda \leq 100$$

(1) for $V_\infty$ = 1 km/sec,

$$0.3R_E \leq r \leq 6250R_E$$

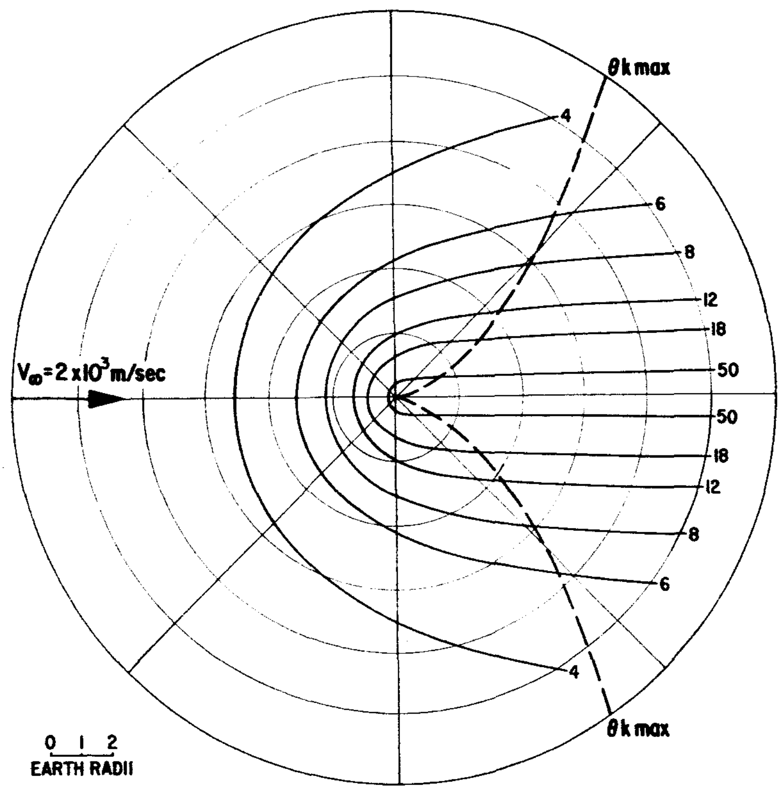


Fig. 4. Total particle flux contours about an infinitesimal attractive center relative to unit monodirectional, monoenergetic incident flux at infinity for $V_\infty = 2 \times 10^3$ m/sec.

which, of course, far exceeds the space within which the earth's field is effectively the only field, and (2) for $V_\infty = 80$ km/sec,

$$5 \times 10^{-5} R_E \leq r \leq 0.98 R_E$$

which is within the earth. However, this deficiency at the higher energies is more apparent than real in that it is evident from Figure 3 that at high energies or great distances ($\lambda > 100$) the flux deviates but very slightly from its value of unity assumed at infinity.

Figure 3 contains a great deal of information about the field. Its main uses can perhaps be best summarized as in Table 2.

*Results and conclusions.* Figures 4 and 5 show a sample set of results obtained by the application of the foregoing analysis to the case of $V_\infty = 2$ km/sec. We recognize that such small relative speeds, if they are physically significant at all, will be so only for the finest (high magnitude) meteoric dust, and we therefore do not insist on their reality. The assumption that $V_\infty = 2$ km/sec was made because these relatively low energy patterns most strongly display the enhancement effect we desired to establish. This effect also exists at higher energies; the higher energy field patterns have been obtained in the accompanying paper [*Hale and Wright,* 1964] where, appropriately, we consider a finite attractive center.

Figure 4 shows the total particle flux contours about an infinitesimal attractive center resulting from a monoenergetic and monodirectional stream incident from the left. Total particle flux means that both scattered and unscattered fluxes have been summed to obtain the final result. Here, as in Figure 5, all flux values have been normalized relative to a value of unity in the incident stream at infinity. The dashed line in the plane of Figure 4 is the $\theta_{k\,\max}$ surface or, rather, the surface generated by the loci of points of perigee; flux and density contours, and

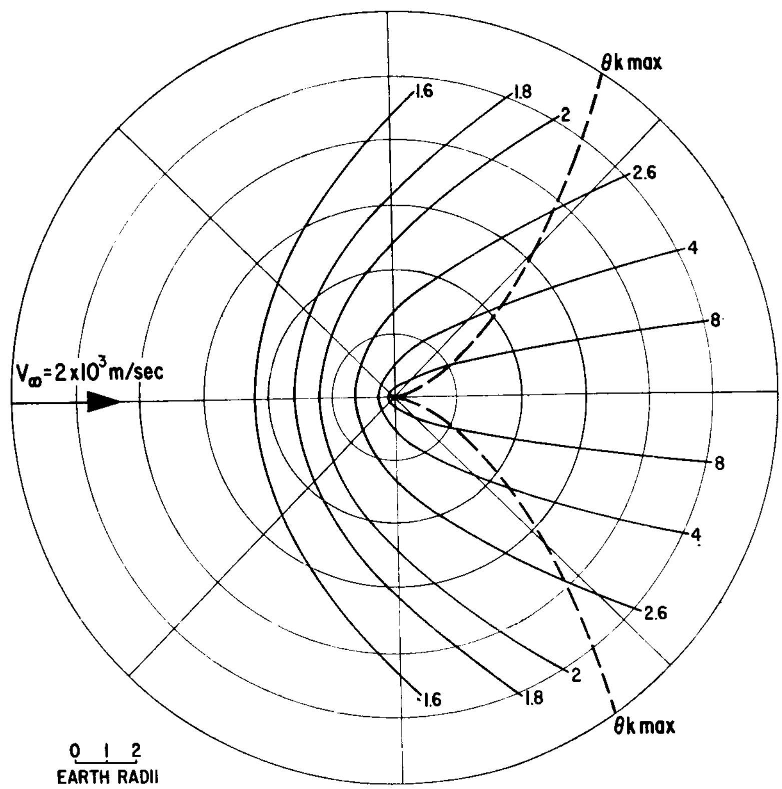


Fig. 5. Total particle density contours about an infinitesimal attractive center relative to a particle density at infinity of $5 \times 10^{-4}$ m$^{-3}$ arriving from a unit monodirectional, monoenergetic flux at infinity for $V_\infty = 2 \times 10^3$ m/sec.

their gradients as well, are continuous upon crossing this surface. It is apparent from Figure 4 that, under the assumed conditions, gravitational focusing can cause localized enhancement of flux by orders of magnitude. This effect is far greater than previously expected, many authorities having felt that enhancement factors might possibly be as great as 2 or 3. Furthermore, we hasten to point out that such enhancement can occur quite close to the earth (earth radii are shown in a scale on Figures 4 and 5) and that an incident stream need be only 5 to 10 earth diameters in width to cause an order of magnitude enhancement.

Figure 5 shows total particle density contours about an infinitesimal attractive center for $V_\infty = 2$ km/sec normalized to unit flux in the undisturbed stream. The enhancement in density is less than that for flux, an effect obviously due to the speeding up of particles as they near the center. As energy increases, the enhancement at any given point of both flux and density decreases, and the flux and density enhancements tend toward unity.

Figure 3 is the universal flux plot discussed in the foregoing section. Essentially, it is an exploitation of the fact that, regardless of the strength of an infinitesimal attractive center or of the magnitude of $V_\infty$, the flux field pattern is unaltered except for a change of scale in distance. The variable $\lambda$, the product of $r$ and $y$, is the basic parameter in this plot. Thus if, at a point distant $r$ from the center in a direction of 150° relative to the radiant of the incident stream, the flux is 6 (corresponding to $\lambda = 0.8$), it is immediately seen from Figure 3 that if the energy is divided by 4 (corresponding to $\lambda = 0.2$) the new flux at this point will be 20 (all flux values assumed unity at infinity).

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(Manuscript received March 9, 1964;
revised June 9, 1964.)